5. Exercise Sheet

Note: Exam level questions are marked as 💡

Exercise 1. Giant component:
Consider a network where all vertices have degree 1 or 4, i.e. $p_k \equiv 0$ for all $k \neq 1, 4$. Compute $g_0(z), g_1(z)$ and the size of the giant component $S$. When does it exist?

Exercise 2. Network valuation:
Consider the network valuation formula

$$E_i = A_i^E + \sum_j A_{ij}^E \mathbb{V}(E_j) - L_i^E - \sum_j L_{ij}^I$$

Now, assume that $\mathbb{V}(E_j) = \alpha E_j$ for some $0 < \alpha < 1$. Without computing anything, interpret the resulting equity values $E$ as node centralities.

Exercise 3. Eisenberg–Noe model:
Consider two banks 1, 2 with external assets $A_1^E = 0.9, A_2^E = 0.8$ and external liabilities $L_1^E = L_2^E = 1$. Show that both banks are insolvent.

Now assume that half of the debt is actually cross-held by the other bank, i.e. $L_{12}^E = L_{21}^E = 0.5$. Compute the resulting bank values under the Eisenberg–Noe model and interpret your findings.

Exercise 4. Site percolation:
Here, we study the resilience of a network to random removal of nodes. To this end, each node is either occupied with probability $\phi$ or removed (not occupied). If $\phi$ is reduced the giant component of the network falls apart into small, disconnected clusters, the percolation transition.

Using the technique of generating functions, derive an expression for the critical value $\phi_c$ corresponding to the percolation transition.
Exercise 5. OEC network of international trade:

- Generate a random bipartite network $M_{cp}$ by randomly matching countries and products, i.e. draw a random incidence matrix where each entry is one with prob. $p$.

- Assume that there are $N_a$ “abilities”. Now, generate a random bipartite network as follows:
  - Randomly assign each country $c$ a set of abilities, e.g. draw a random binary $N_c \times N_a$ matrix where each entry is one with prob. $p_1$.
  - Randomly assign each product $p$ a set of abilities (with prob. $p_2$).
  - There is an edge between $c$ and $p$ iff country $c$ has all abilities required to produce product $p$.

Compute and plot $k_{c,1}$ over $k_{c,0}$ for both models (with varying parameters):

- Which model is more realistic?

- How could you further improve the model?