Exercise 1. Sparsity priors provide an alternative to model selection. Study the example from the sklearn documentation https://scikit-learn.org/stable/auto_examples/linear_model/plot_ard.html.

1. Explain the automatic relevance determination (ARD) model.
2. How can it be used instead of model selection?

3 points

Exercise 2. Consider the Gaussian mixture model with \( K = 2 \)

\[
p(x) = \sum_{k=1}^{K} p(\mu_k) N(x|\mu_k, \sigma_k^2)
\]

where \( x \) can be assumed scalar.

1. Argue that the maximum likelihood estimate is not defined for this model.
2. What about the Bayesian posterior?

3 points


1. Fit the provided Stan model on the extended data set.
2. Compare it to other models you consider reasonable. Can you improve the model?

4 points