Exercise 1. Shortest distance and breadth-first search

1. Illustrate the breadth-first search by determining the shortest distance between vertex 2 and all other vertices in the graph $G$ displayed in Fig. 1. In which order are the vertices visited by the algorithm?

Exercise 2. Directed graphs

Find a directed graph of at least six vertices with the following properties:

- Exactly two weakly connected components
- At least three strongly connected components (SCC).

Find the in- and out-components of all your SCC.
Exercise 3. Topological sorting

Implement an R function with inputs
- a graph $G$ (in igraph format)
- a vertex $v \in V(G)$

and outputs
- a vector containing all vertices of $G$ in topological order

Note:
- You may want to start by writing down an algorithm in pseudo-code
- Your function should throw an error if $G$ is not a DAG
  
  Hint: `stop("This should not happen!")` raises an error.
- Analyse the runtime of your algorithm:
  - Would your function work on very large graphs?

Exercise 4. Power laws

Download the Amazon product co-purchasing network, March 02 2003 data from the Stanford Large Network Dataset Collection [http://snap.stanford.edu/data/amazon0302.html](http://snap.stanford.edu/data/amazon0302.html)

1. Use `read_delim` and `graph_from_data_frame` to load the data into R.
   
   Note: Inspect the data file and pass suitable arguments to `read_delim`

2. Compute and plot the degree distribution of the graph.

3. Install the `poweRlaw` library and look at its documentation:
   - Fit the tail of the degree distribution with a discrete power law distribution?
     Which lower cutoff value $x_{\text{min}}$ would you choose?
   - Compare your fit with an exponential and log-normal distribution. Do you (still) believe that the data are distributed according to a power law?